Evaluation of the Effect of Stray Current Collection System in DC-Electrified Railway System

Wei Liu, Member, IEEE, Tian Li, Jie Zheng, Weiguo Pan, and Yichen Yin

Abstract—Stray current corrosion damage is significant in DC traction power supply system (DTS). Stray current collection system including stray current collection network (SCCN) and stray current drainage device (SCDD) is designed to control the stray current. A comprehensive system integrated calculation model is established to evaluate the stray current. And the iterative power flow algorithm with simplification of DTS is proposed. The stray current distribution is calculated by multi sections rail reflux system model. The model is verified with CDEGS simulation. The influence of stray current collection system on stray current is evaluated. The result shows that the primary stray current is mainly affected by rail to SCCN resistance and the length of the line. The efficiency of collecting stray current of SCCN is mainly affected by the resistance of SCCN and the SCCN to earth resistance. While the use of SCDD deteriorates the efficiency of collecting stray current of SCCN. A site test was carried out in Chengdu Metro Line 1 to evaluate the effect of stray current collection system.

Index Terms—DC traction power supply system, stray current, stray current collection network, stray current drainage device.

I. INTRODUCTION

ORDINARILY, the rails are used as return paths for traction current in DC traction systems (DTS). Low rail-to-earth conductivity exists in the insulated and dry environment. Actually, part of traction current would leak through the rail and is called stray current. Stray current can not only increase the corrosion damage of metal constructions, such as pipelines [1], but also cause DC bias of transformers [2]. The magnitude of stray current depends on the distribution of rail potential.

In order to study the stray current, some scholars have carried out different modeling analysis in [3]–[7] in order to investigate the generation of stray current and its impact on DTS. A π-type model is established in [3], and the influence of over zone feeding on rail potential is analyzed. In addition, the lumped parameter model does not consider the details of rail potential distribution. Thus, most of the models are based on the micro element model, solved by Kirchhoff’s law and boundary conditions. In [4], the influence of traction characteristics and traction path of metro vehicles on stray current is studied. In [5], how the stray current magnitude varies according to the design of the power system and the running rails is illustrated, and the influence of rail to ground transition resistance on stray current is discussed, which is also mentioned in [6],[7]. The difference is that the rail to earth resistance is uniformity or the rail to earth resistance does not affect distribution characteristics of stray current. Besides, insulation defects and the location of insulation defects are discussed respectively in [6].

The application and analysis of stray current distribution models are generally considered in the power supply section between the two traction power substations (TPSs). In fact, due to the electrical connection of the rails along the line, stray current leakage occurs anywhere in the rails. The electrical connection between lines should also be considered. In [8], it has been proved that cancelling the connection between lines can reduce the stray current.

Other studies conducted analysis on the main causes of stray current. These studies focus on the earthing mode of TPSs, vehicle status, civil construction of tunnels, track to earth resistance and so on [5]–[11]. The main strategies of reducing stray current include shortening the distance of substation, increasing the rail to earth resistance, reducing the resistance of reflux system and increasing cross section of the additional reinforcement earthing conductor [12]. Most systems are installed with the stray current collection networks (SCCN). Due to the existence of the SCCN, the leakage of stray current to the earth is reduced. The efficiency of collecting stray current by SCCN should also be concerned.

Previous studies have reported that the installation of the SCCN cannot reduce primary stray currents, but will reduce secondary stray current [13]. The closer the SCCN to the rail is, the wider the incidences of stray current collection are [14]. Some investigators considered the effects of soil structure and SCCN structure on stray current. The efficiency of collecting stray current of SCCN is 86.32% in the finite element simulation when the resistivity of the concrete is 1000 Ω·m and the resistivity of lower soil is 300 Ω·m [15].

Among several earthing systems, the floating system is best for DTS to reduce corrosion risk [5], [16]. In China, floating system is widely applied, and each TPS is installed with stray current drainage devices (SCDDs), while the SCDD is seldom used in European countries. Relatively few studies concentrated on the effect of the SCDD. KieBling, etc, have pointed out that the SCDD would double the rail potential and quadruple the stray current in the floating earth system [17]. The operation of SCDDs increases the rail potential significantly in [18]. In China, the SCDD is in an awkward situation now. It is designed...
and installed in each DTS, but it is not put into operation even if stray current is already serious in the existing metro lines. The contributions of this paper are as follows.

1) A comprehensive system integrated calculation model of DC system is established to evaluate the stray current. Considering the coupling relationship between traction network and reflux system, the iterative power flow algorithm with simplified the network of DTS is proposed. In addition, the rail potential and stray current distribution are obtained by the multi sections rail reflux system model. The continuous condition of voltage and current is used for solving K-coefficients of multiple sections.

2) Considering the stray current collection system, the influence of stray current collection system parameters and SCDD operation on stray current is evaluated, which guides the design of stray current protection system. In Chengdu Metro Line 1, a single train operation experiment is used to evaluate the stray current distribution and the stray current collection efficiency of SCDD.

The rest of this article is organized as follows. In Section II, the stray current collection system is described in detail. In Section III, a comprehensive system integrated calculation model is proposed. In Section IV, a comparison between the analytical calculation model and CDEGS simulation model of multi sections rail reflux system is made. And the effect of the rail reflux system parameters on stray current is evaluated. In Section V, the distribution of dynamic rail potential and stray current is verified by a single train operation test in Chengdu Metro Line 1 in China. Finally, the conclusions are presented in Section VI.

II. DESCRIPTION OF STRAY CURRENT COLLECTION SYSTEM

Stray current corrosion damage is significant among the investigated features of the earthing systems. The floating system is the best owing to the small amount of rail potential hazard [16], which has no intentional connection to earth as shown in Fig. 1. SCCN consists of structural steel bars and is embedded in bed blocks. SCCN between different bed blocks is connected by 95 mm² cable through the connection terminal. And the earthing nets under each TPS are connected by an earth wire.

The stray current leaked to the earth is indeed small due to the good insulation performance of rail fastener. The accumulation of pollution over time has a negative effect on insulation performance and stray current increases. If SCCN does not exist, the stray current leaks directly into the earth, causing corrosion of metal pipeline and metal structure. The SCCN can provide additional paths for primary stray current what are from rail to TPS via SCCN and can reduce the stray current leaks into underground. In other words, when there has SCCN, stray current leaks from rail to SCCN, and then returns to TPSs in two paths. The current which leaks from rail is called primary stray current, and the current which is not directly collected by SCCN is called secondary stray current.

The original purpose of installing the SCDD in TPS in China to provide a low resistance path for current to flow back to the TPS directly. Whether the SCDD can deteriorate the stray current has been a dispute in DTS. As shown in Fig. 1, SCCN branch of the SCDD is a drainage diode connected the SCCN to the main circuit of the SCDD. Part of the stray current collected by the SCCN is returned to the negative bus of TPS through the SCCN branch. And earth branch of the SCDD is diode connected from the TPS earthing net to main circuit of the SCDD. The stray current absorbed by buried metal structure is collected by earth branch. There is a current limiting resistor $R_l$ in series with the main circuit of the SCDD. The purpose is to prevent excessive stray current from damaging the drainage diodes. The resistance is usually set as 0 to 5 Ω.

There are two kinds of operation modes of the SCDD: automatic mode and manual mode. In the automatic mode, the SCDD is controlled by the stray current monitor system, and the polarization potential of reinforcement in SCCN is the monitoring object. In the manual mode, the control pattern can be locally or remotely control. However, SCDDs at TPSs barely work because of the lack of operation criterion and concern of aggravation of stray current.

III. COMPREHENSIVE SYSTEM INTEGRATED CALCULATION MODEL

A. Single Section Reflux System Model

The distributed micro element network model between $x_1$ and $x_2$, called “M” network, is shown in Fig. 2. $i_s(x)$ is the current in rail and $i_s(x)$ is the current in SCCN. $u_{rs}(x)$ represents rail to SCCN potential, and $u_{sc}(x)$ represents SCCN to earth potential. It is assumed that longitudinal resistance of the rail $R_{rs}$, longitudinal resistance of the SCCN $R_{sc}$, the rail to SCCN resistance $R_{rs}$ and the SCCN to earth resistance $R_{se}$ are equivalent uniform parameters. All these parameters change within the allowable range according to the practical situation [19].
The lumped parameter model is shown in Fig. 3. The distributed circuit is equivalent to II lumped circuit. \( r_1 \) is the equivalent longitudinal resistance of rail between \( x_1 \) and \( x_2 \). \( z_{sl} \) is the equivalent longitudinal resistance of SCCN between \( x_1 \) and \( x_2 \). \( y_{rs} \) is the equivalent rail to SCCN resistance between \( x_1 \) and \( x_2 \). \( y_{sc} \) is the equivalent SCCN to earth resistance between \( x_1 \) and \( x_2 \).

Take the current to the right as the positive direction, the “M” network can be expressed with (1) by Kirchhoff’s law, and the potential and current can be expressed by (2). \( \mathbf{H} \) and \( \mathbf{H}^{-1} \) can be expressed as (3) and (4).

\[
\begin{align*}
\mathbf{u}_s &= i_s R_s \mathbf{a} - i_R \mathbf{a} \mathbf{x} \frac{d}{dx} \mathbf{u}_s = -i_s R_s \mathbf{a} \\
i_r(x) &= i_r \mathbf{a} \mathbf{x} \frac{d}{dx} u_r(x) \mathbf{u}_s &= u_r(x) \mathbf{a} \mathbf{x} \frac{d}{dx} u_s(x) - u_s(x) \mathbf{a} \mathbf{x} \frac{d}{dx} u_s(x) \\
\mathbf{H} &= \begin{bmatrix} 1 & 1 & 1 & 1 \\
d_4 & 1 & 1 & 1 \\
d_3 & -1 & 1 & 1 \\
d_2 & -1 & -1 & 1 \\
d_1 & -1 & -1 & -1 \\
d_1 & 1 & 1 & 1 \\
d_2 & -1 & 1 & 1 \\
d_3 & -1 & -1 & 1 \\
d_4 & 1 & 1 & -1 \end{bmatrix} \\
\mathbf{H}^{-1} &= \begin{bmatrix} h_1' \\
h_2' \\
h_3' \\
h_4' \end{bmatrix} = \begin{bmatrix} h_1' \\
h_2' \\
h_3' \\
h_4' \end{bmatrix} + \begin{bmatrix} h_1'' \\
h_2'' \\
h_3'' \\
h_4'' \end{bmatrix} \begin{bmatrix} h_1'' \\
h_2'' \\
h_3'' \\
h_4'' \end{bmatrix}
\end{align*}
\]

where \( k_1, k_2, k_3, k_4 \) are called \( \mathbf{K} \) coefficients, which are determined by boundary conditions between two sections. Others are known parameters, denoted by the following.

\[
\begin{align*}
\lambda_1 &= \sqrt{b_1 + (b_2 - 4 R_c R_s / (R_{rs} R_{sc}))^2 / 2} \sqrt{b_2 - 4 R_c R_s / (R_{rs} R_{sc})} \\
\lambda_2 &= \sqrt{b_1 + (b_2 - 4 R_c R_s / (R_{rs} R_{sc}))^2 / 2} \sqrt{b_2 - 4 R_c R_s / (R_{rs} R_{sc})} \\
b &= (R_{rs} R_{sc} R_{rs} R_{sc} / (R_{rs} R_{sc})) \sqrt{b_2 - 4 R_c R_s / (R_{rs} R_{sc})} \\
d_1 &= (R_{rs} R_{sc} R_{rs} R_{sc} / (R_{rs} R_{sc})) \sqrt{b_2 - 4 R_c R_s / (R_{rs} R_{sc})} \\
d_2 &= (R_{rs} R_{sc} R_{rs} R_{sc} / (R_{rs} R_{sc})) \sqrt{b_2 - 4 R_c R_s / (R_{rs} R_{sc})} \\
f_1 &= -\lambda_1 R_{rs} \\
f_2 &= -\lambda_2 R_{rs} \\
d_1 &= -\lambda_1 R_{rs} R_{sc} (R_{rs} + R_{rs} / \lambda_1 R_{rs}) \\
d_2 &= -\lambda_2 R_{rs} R_{sc} (R_{rs} + R_{rs} / \lambda_2 R_{rs}) \end{align*}
\]

When the reflux system is expressed as distributed model as Fig. 2, it is satisfied with (2) at \( x_1 \) and \( x_2 \). Finally, the relationship can be expressed as (5).

\[
\begin{align*}
\begin{bmatrix} i_r(x_1) \\
i_s(x_1) \\
u_{rs}(x_1) \\
u_{se}(x_1) \end{bmatrix} &= \mathbf{H} \begin{bmatrix} i_r(x_2) \\
i_s(x_2) \\
u_{rs}(x_2) \\
u_{se}(x_2) \end{bmatrix} \\
\begin{bmatrix} i_r(x_1) \\
i_s(x_1) \\
u_{rs}(x_1) \\
u_{se}(x_1) \end{bmatrix} &= \mathbf{H} \begin{bmatrix} i_r(x_2) \\
i_s(x_2) \\
u_{rs}(x_2) \\
u_{se}(x_2) \end{bmatrix} \end{align*}
\]

From (5), \( i_s(x_2) \) can be described as (6) and (7).

\[
\begin{align*}
i_r(x_2) &= c_1 i_r(x_1) + c_2 i_s(x_1) + c_3 u_{rs}(x_1) + c_4 u_{se}(x_1) \\
\begin{bmatrix} c_1 \\
c_2 \\
c_3 \\
c_4 \end{bmatrix} &= \begin{bmatrix} h_1' \lambda_{s1} + h_1' e^{-\lambda_{s1}} x_1 \\
h_1'' \lambda_{s1} x_1 + h_1'' e^{-\lambda_{s1}} x_1 \\
h_1' \lambda_{s1} x_1 + h_1' e^{-\lambda_{s1}} x_1 \\
h_1'' \lambda_{s1} x_1 + h_1'' e^{-\lambda_{s1}} x_1 \end{bmatrix} \\
\begin{bmatrix} h_1' \lambda_{s1} + h_1' e^{-\lambda_{s1}} x_1 \\
h_1'' \lambda_{s1} x_1 + h_1'' e^{-\lambda_{s1}} x_1 \\
h_1' \lambda_{s1} x_1 + h_1' e^{-\lambda_{s1}} x_1 \\
h_1'' \lambda_{s1} x_1 + h_1'' e^{-\lambda_{s1}} x_1 \end{bmatrix} \end{align*}
\]

When the circuit is expressed as lumped circuit as Fig. 3, the relationship between \( i_r(x_2) \) and \( i_r(x_1) \), \( i_s(x_1) \), \( u_{rs}(x_1) \), \( u_{se}(x_1) \) can be expressed as (8).

\[
\begin{align*}
i_r(x_2) &= (1 + y_{rs} + z_{sl} i_r(x_1)) - (y_{rs} + z_{sl} i_s(x_1)) i_r(x_1) \\
&+ y_{rs} (2 + y_{rs} + z_{sl} + y_{rs} + z_{sl} + u_{rs}(x_1)) u_{rs}(x_1) \\
&+ (z_{sl} + y_{se} + y_{rs}) u_{se}(x_1) \end{align*}
\]

So the equivalent lumped parameters of the reflux system can be expressed as (9) according to (6) and (8), where \( c_1 - c_4 \) is only determined by \( \mathbf{H} \) and distance between two sections.

\[
\begin{align*}
z_{sl} &= (1 - c_1) / c_3 \\
y_{rs} &= c_3 / c_2 \\
y_{se} &= c_2 / c_1 \end{align*}
\]
constant powers $P_k$, $B$ and $E$ are the ends of the line at 0 and 1 respectively. Along the rail, the position of the trains, the TPSs and the ends are divided into $m+n+2$ sections (Section 0 to Section $m+n+1$).

In Fig. 4, $z_{ij}$ ($j = 1, 2, 3 \ldots m+n+1$) is the resistance of traction network. $z_{dej}$ is the additional connection resistance between “$r$” and “$s$” or “$r$” and “$e$”. In TPS, when the SCDD works and conducts, $z_{dej} = z_{dej} = 2R$, otherwise $z_{dej} = z_{dej} = \infty$.

The lumped parameters model of the DTS and reflux system is described as shown in Fig. 5. The TPS is equivalent to the Norton’s theorem. “M” network is equivalent to lumped parameter according to III.A.

The whole network can be expressed as node voltage analysis method as (10), as shown at the bottom of this page.

Where, node injection current $I_{trs-j} = [I_{ij} \ I_{rj} \ I_{sj}]^T$, node voltage $U_{trs-j} = [U_{ij} \ U_{rj} \ U_{sj}]^T$, $Y_{trs-j}$ is the self-admittance matrix and others are the mutual admittance matrix. When $j = 1$ to $m+n$, $Y_{trs-j(1)}$, $Y_{trs-j(j)}$, $Y_{trs-j(j+1)}$ can be described by (11) to (14).

$$Y_{trs-j} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & B_{23} \\ 0 & B_{32} & B_{33} \end{bmatrix}$$

$$Y_{trs-j(j-1)} = \begin{bmatrix} -1/z_{tj} & 0 & 0 \\ 0 & -1/z_{rj} & 0 \\ 0 & 0 & -1/z_{sj} \end{bmatrix}$$

$$Y_{trs-j(j+1)} = \begin{bmatrix} -1/z_{t(j+1)} & 0 & 0 \\ 0 & -1/z_{r(j+1)} & 0 \\ 0 & 0 & -1/z_{s(j+1)} \end{bmatrix}$$

Noting that if the section $j$ is not the TPS section, $z_{pj} = \infty$, otherwise $z_{pj}$ is internal resistance of TPS.

For the “$t$” nodes, $I_t = [I_{t0} \ I_{t1} \ \ldots \ I_{t(m+n+1)}]^T$, $U_t = [U_{t0} \ U_{t1} \ \ldots U_{t(m+n+1)}]^T$. For the “$r$” nodes, $I_r = [I_{r0} \ I_{r1} \ \ldots I_{r(m+n+1)}]^T$, $U_r = [U_{r0} \ U_{r1} \ \ldots U_{r(m+n+1)}]^T$. For the “$s$” nodes, $I_s = [I_{s0} \ I_{s1} \ \ldots I_{s(m+n+1)}] = [0]$, $U_s = [U_{s0} \ U_{s1} \ \ldots U_{sn}]$.

When combining all kinds of nodes together, (10) can be expressed as (15). Where $Y_{11}$, $Y_{12}$, $Y_{21}$, $Y_{22}$, $Y_{23}$, $Y_{32}$, $Y_{33}$ are the elements of the admittance matrix.

$$\begin{bmatrix} I_t \\ I_r \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & 0 \\ Y_{21} & Y_{22} & Y_{23} \\ 0 & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} U_t \\ U_r \\ U_s \end{bmatrix}$$

The purpose of iterative power flow algorithm is to obtain the current of TPSs and trains through the voltage between “$t$” nodes and “$r$” nodes. It can be found that $I_t = -I_r$. After simplified the DC traction network, DTS and reflux system are merged as Fig. 6. The voltage of $j$th merged node is $U_{tjr} = U_{tj} - U_{rj}$. The current of $j$th node is $I_{tj}$.

After simplification, the network can be expressed as (16).

$$U_{tr} = U_t - U_r = Y_{eq} I_t$$

$$I_{tr-s} = \begin{bmatrix} I_{trs-0} \\ \vdots \\ I_{trs-j} \\ \vdots \\ I_{trs-(m+n+1)} \end{bmatrix}, \quad Y_{trs-j} = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 0 & Y_{trs-j(j-1)} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix}, \quad U_{trs-j} = \begin{bmatrix} U_{trs-0} \\ \vdots \\ U_{trs-j} \\ \vdots \\ U_{trs-(m+n+1)} \end{bmatrix}$$

$$Y_{eq} = \begin{bmatrix} Y_{trs-(m+n+1)(m+n+1)} \end{bmatrix}$$
TABLE I
ITERATIVE POWER FLOW ALGORITHM WITH SIMPLIFICATION OF DTS

| Input: | nodes and branches of the lumped parameter model, power of the trains \( \mathbf{P} = [P_{t_1}, P_{t_2}, \ldots, P_{t_n}] \), voltage source of TPSs \( \mathbf{U}_p = [U_{p_1}, U_{p_2}, \ldots, U_{p_m}] \), internal resistance of TPSs \( \mathbf{z}_p = [z_{p_1}, \ldots, z_{p_i}, \ldots, z_{p_m}] \), of parameters of SCDDs, convergence tolerance \( \varepsilon \) |
| Output: | \( U_{tr} \) |

Steps:
1. Set \( \text{it}=1 \rightarrow \) Number of iterations
2. Initialize \( \forall k, \quad U_r^{(1)} = U_{tr} \leftarrow \text{Noload voltage}, \quad U_r^{(0)} = 0 \)
3. Compute \( Y_{eq} \)
4. Compute \( \forall k, f_{rk}^{(0)} = P_i / (U_r^{(0)} - U_r^{(2)}) \)
5. According to (16), compute \( U_p^{(0)} \)
6. Compute \( \text{error} = \max(|U_p^{(0)} - U_p^{(0-1)}|) \)
7. If \( \text{error} > \varepsilon \) goto 4, \( \text{it}=\text{it}+1 \)
8. Else break

Where \( Y_{eq} = -B^{-1}(E + AY^{-1}) \), \( Y_{12} = [E + Y_{11}B^{-1}(E + AY^{-1})] \), \( E = -Y_{21}AY_{12}^{-1} + Y_{22}Y_{33}^{-1} \), \( I \) is identity matrix.

At time stamp \( t \), the iterative power flow algorithm with simplification of DTS is shown in Table I.

C. Multi Sections Rail Reflux System Model

\( U_{tr} \) is obtained by iterative power flow algorithm with simplification of DTS. Then, the current and voltage distribution along the rail can be calculated by the following boundary conditions.

There is no current injection at two ends of the line in floating earth system. The boundary condition at \( B \) and \( E \) can be expressed as (17).

\[
\begin{align*}
    &i_r(0) = 0, \quad i_s(0) = 0 \\
    &i_r(l) = 0, \quad i_s(l) = 0
\end{align*}
\]

If \( k \)th train is at \( j \)th section, \( i_r(y_k), i_s(y_k), u_{rs}(y_k), u_{sc}(y_k) \) can be calculated by (2) from \( (j-1) \)th section to \( j \)th section, which is \( i_r(x_{i+}), i_s(x_{i+}), u_{rs}(x_{i+}), u_{sc}(y_{i+}) \). Also \( i_r(x_{i-}), i_s(x_{i-}), u_{rs}(x_{i-}), u_{sc}(y_{i-}) \) can be calculated by (2) from \( (j+1) \)th section to \( j \)th section, which is \( i_r(x_{i-}), i_s(x_{i-}), u_{rs}(y_{i-}), u_{sc}(y_{i-}) \).

The train current \( I_k \) is obtained according to equation (18). The potential of rail to SCCN and SCCN to earth are continuous which are expressed in (19).

\[
I_k = P_{tk} / u_{trj}
\]

\[
\begin{align*}
    &u_{rs}(y_{k-}) = u_{rs}(y_{k+}), \quad u_{sc}(y_{k-}) = u_{sc}(y_{k+}) \\
    &i_r(y_{k-}) - i_r(y_{k+}) = -I_k, \quad i_s(y_{k-}) - i_s(y_{k+})
\end{align*}
\]

If \( i \)th TPS is at \( j \)th section, \( i_r(x_i), i_s(x_i), u_{rs}(x_i), u_{sc}(x_i) \) can be calculated by (2) from \( (j-1) \)th section to \( j \)th section, which is \( i_r(x_{i-}), i_s(x_{i-}), u_{rs}(x_{i-}), u_{sc}(x_{i-}) \). Also \( i_r(x_{i+}), i_s(x_{i+}), u_{rs}(x_{i+}), u_{sc}(x_{i+}) \)

Fig. 7 Integrated calculation flow of power flow and distribution.

\[
\begin{align*}
    &u_{rs}(x_i) < -U_d (22) \\
    &u_{rs}(x_i) = u_{rs}(x_i) + 2U_r + i_r(x_i) - i_r(x_i +) = J_i \\
    &u_{rs}(x_i) = u_{rs}(x_i) + 2U_r + i_r(x_i) - i_r(x_i +) = J_i
\end{align*}
\]

If the SCDD works and diode conducted as (22), ignoring the diode forward voltage, the current return to the TPS through the SCDD can be determined as \( -u_{rs}(x_i) / R_s/u_{sc}(x_i) / R_s \). Conditions can be expressed with (23).

If the SCDD does not work, conditions can be expressed with (21).

\[
\begin{align*}
    &J_i = U_{p_i} / z_{pi} - u_{trj} / z_{pi} \\
    &i_r(x_i) - i_r(x_i +) = J_i, \quad i_s(x_i) = i_s(x_i +) \\
    &u_{rs}(x_i) = u_{rs}(x_i +), \quad u_{sc}(x_i) = u_{sc}(x_i +)
\end{align*}
\]

IV. CASES VERIFICATION AND ANALYSIS

The comprehensive system integrated calculation model considering the SCDD is verified with CDEGS simulation model. Then, the influence of rail reflux system parameters on stray current is analyzed.

A. The CDEGS Model of Multi Sections Rail Reflux System

The multi sections rail reflux system model verification is carried out by taking account of 7 TPSs and 6 trains as Fig. 7. The length of the line \( l = 20.00 \) km. Two cases are considered. Case 1 is all the SCDDs under non-working situation. Case 2 is the SCDD of S1 under working situation. \( R_s \) is set to 0.1 \( \Omega \) here.
The CDEGS model is shown in Fig. 8. Considering soil stratification, the whole space is vertically divided into four layers. They are air, soil-1, concrete, soil-2 layers from top to bottom. Rails and SCCN are placed in the concrete layer. Due to the existence of cross bond between rails, the rails can equal to one conductor with a radius of 49.00 mm [6]. Rail fasteners are evenly distributed along the rail, so 10 mm coating with $10^5 \, \Omega \cdot m$ resistivity can be uniformly set on the surface of the conductor to simulate the influence of rail to ballast bed transit resistance. The SCCN is equivalent to a steel reinforce bars under rails [21]. Because of the lack of diode model in CDEGS, connection wires with current limiting resistances are used to simulate the SCDDs. TPSs are replaced by powers supply with the phase angle of 180°, and similarly, the trains are replaced by powers supply with the phase angle of 0°.

The method described in appendix A.2 of IEC 62128-2 can be used to calculate rail to SCCN and SCCN to the earth resistance equivalently. Parameters of the CDEGS model and the analytical calculation model of the multi sections rail reflux system are set as Table II.

The CDEGS model is shown in Fig. 8. Considering soil stratification, the whole space is vertically divided into four layers. They are air, soil-1, concrete, soil-2 layers from top to bottom. Rails and SCCN are placed in the concrete layer. Due to the existence of cross bond between rails, the rails can equal to one conductor with a radius of 49.00 mm [6]. Rail fasteners are evenly distributed along the rail, so 10 mm coating with $10^5 \, \Omega \cdot m$ resistivity can be uniformly set on the surface of the conductor to simulate the influence of rail to ballast bed transit resistance. The SCCN is equivalent to a steel reinforce bars under rails [21]. Because of the lack of diode model in CDEGS, connection wires with current limiting resistances are used to simulate the SCDDs. TPSs are replaced by powers supply with the phase angle of 180°, and similarly, the trains are replaced by powers supply with the phase angle of 0°.

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### Table II

<table>
<thead>
<tr>
<th>The CDEGS model parameters</th>
<th>The analytical calculation model parameters</th>
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<tbody>
<tr>
<td>Soil-1 resistivity</td>
<td>$38 , \Omega \cdot m$</td>
</tr>
<tr>
<td>Soil-2 resistivity</td>
<td>$38 , \Omega \cdot m$</td>
</tr>
<tr>
<td>Concrete resistivity</td>
<td>$250 , \Omega \cdot m$</td>
</tr>
<tr>
<td>10mm coating resistivity</td>
<td>$10^5 , \Omega \cdot m$</td>
</tr>
<tr>
<td>Rail resistivity</td>
<td>$36.5 , m\Omega/km$</td>
</tr>
<tr>
<td>A steel reinforce bar in SCCN resistivity</td>
<td>$0.20 , \Omega/km$</td>
</tr>
<tr>
<td>$R_{se}$</td>
<td>$4.102 , \Omega \cdot km$</td>
</tr>
<tr>
<td>$R_{se}$</td>
<td>$0.190 , \Omega \cdot km$</td>
</tr>
<tr>
<td>$R_{ss}$</td>
<td>$18.25 , m\Omega/km$</td>
</tr>
<tr>
<td>$R_{bl}$</td>
<td>$0.204 , \Omega/km$</td>
</tr>
</tbody>
</table>

The comparative analysis of simulation results

The comparison between simulation of the CDEGS model and the analytical calculation model of the rail reflux system model are shown in Fig. 9 to Fig. 10.

It can be seen from Fig. 9 that the errors of rail current $i_r$ is less than 6.41 A in Case1, which is 0.42% of the CDEGS result respectively. And error of $i_r$ is less than 26.70A in Case2, which is 1.69% of the CDEGS result respectively.

As shown in Fig. 10, the rail to SCCN potential of CDEGS simulation is coincident well with the analytical calculation model result. The average errors of $u_{rs}$ between the analytical calculation result and CDEGS simulation result are less than 0.01 V under Case1 and 0.433V under Case2. All of the errors are less than 1.58% of the highest potential in each case.

Because the SCDD of S1 works, the $u_{rs}$ rises to near 0V at S1. The longitudinal rail potential drop is almost the same, so the average $u_{rs}$ increases 10.35 V.

In fact, $u_{rs}$ is related to $K$ coefficients. In the analytical calculation model, the line is divided into 14 sections. The 14×4
$K$ coefficients can be worked out. $K_1-K_4$ and $K'_1-K'_4$ are the $K$ coefficients between Section 0 and Section 1 under Case1 and Case2 respectively. They are as shown in Table III. Due to the relationship of (27), $I_{rs}$ between Section 0 and Section 1 in Case2 is greater than Case1.

$$\begin{align} K'_1 - K_1 & < 0 \\ K'_2 - K_2 & > 0 \\ K'_3 - K_3 & < 0 \\ K'_4 - K_4 & > 0 \end{align}$$ (27)

The analytical calculation and CDEGS simulation results of stray current are shown in Table IV. In Case1, the difference of $I_{rs}$ is only 0.15A. Since the concrete layer and the soil layer are infinite in the CDEGS model, the difference of SCCN to the earth potential results in the difference of $I_{se}$ of 0.57A. The distribution difference between analytical calculation and CDEGS simulation is acceptable. At this time, the value of $I_{rs}$ as denominator is small, resulting in an efficiency of collecting stray current error about 2.24%. In Case2, the error of $I_{rs}$ is 1.89 A, and the error of $I_{se}$ is 1.11A. The error of $\eta$ is only 0.49%.

Compared with Case1, the $I_{rs}$ in Case2 increases by 2.43 times of analytical calculation and 2.52 times of CDEGS simulation. The SCDD working increases the leakage of stray current in anode section. The $I_{se}$ in Case2 increases by 4.12 times of analytical calculation and 4.45 times of CDEGS simulation. The increase ratio of $I_{se}$ is greater than the increased ration of $I_{rs}$, so $\eta$ is reduced. The $\eta$ in Case2 is 0.29 times of analytical calculation and CDEGS simulation. Most of the leakage stray current is collected by the earth branch, which is the secondary stray current.

### C. Influence of Rail Reflux System Parameters

Various rail reflux system parameters are evaluated by changing a parameter in the appropriate range and other parameters are set as Table II.

The rail fastener is an important component in the rail reflux system. The insulation performance and dirt of rail fastener affect $R_{rs}$. The $I_{rs}$ and $\eta$ changing with $R_{rs}$ in Case1 and Case2 are shown in Fig. 11.

In two cases, $I_{rs}$ and $\eta$ are decreased significantly, and tend to stable with the increase of $R_{rs}$. The greater the $R_{rs}$, the less the $I_{se}$. In Case1, $R_{se} = 0.23 \, \Omega \cdot \text{km}$, $R_{rs} = 18.25 \, \Omega / \text{km}$, $R_{se} = 0.20 \, \Omega / \text{km}$, when $R_{rs} > 15.00 \, \Omega / \text{km}$, $I_{rs}$ is less than 6.16 A, $\eta$ is close to 64.25%. When the rail to SCCN resistance keeps high, about two-third of the stray current can be collected by the SCCN. Nevertheless, it is only 14.73% in Case2. The SCDD working reduces the efficiency of collecting stray current of SCCN. When $R_{rs} = 15.00 \, \Omega / \text{km}$, $I_{rs}$ is up to 18.22A in Case2, which is 2.96 times as much as Case 1. $I_{se}$ of Case2 is 7.05 times as much as Case1.

The SCCN is installed in the concrete ballast bed. The concrete resistivity depends on the pore structure and the pore fluid chemicals of concrete. Different concrete production processes, such as water-binder ratio, admixture, etc., change the $R_{se}$. The variation of $I_{rs}$ and $\eta$ with $R_{se}$ are shown in Fig. 12. The results show that the increase of $R_{se}$ causes a slow decrease of $I_{rs}$ and significant increase of the $\eta$. When $R_{se}$ is larger than 1.0012 km, $\eta$ can be increased to 82.38%. Maintaining a high $R_{se}$ can increase the efficiency of collecting stray current of SCCN, subsequently reduce the $I_{se}$ and the pipeline corrosion. In case 2, $\eta$ is less than 30.00%.

95 mm$^2$ cables is utilized to connect the connecting terminals between ballast beds. Electrical connectivity, cross-section size of connection cables and steel reinforce bars in SCCN affect the

### Table III

**RESULT OF $K$ COEFFICIENTS UNDER DIFFERENT CASES**

<table>
<thead>
<tr>
<th>Case1</th>
<th>Case2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>0.006</td>
</tr>
<tr>
<td>$K_2$</td>
<td>-0.006</td>
</tr>
<tr>
<td>$K_3$</td>
<td>28.890</td>
</tr>
<tr>
<td>$K_4$</td>
<td>-28.890</td>
</tr>
</tbody>
</table>

### Table IV

**RESULTS OF STRAY CURRENT**

<table>
<thead>
<tr>
<th>Case</th>
<th>Analytical calculation</th>
<th>CDEGS simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{se}(A)$</td>
<td>$I_{se}(A)$</td>
</tr>
<tr>
<td>1</td>
<td>22.21</td>
<td>11.18</td>
</tr>
<tr>
<td>2</td>
<td>53.97</td>
<td>46.07</td>
</tr>
</tbody>
</table>

**Fig. 11.** Variation of $I_{rs}$ and $\eta$ with $R_{rs}$ in different cases.

**Fig. 12.** Variation of $I_{rs}$ and $\eta$ with $R_{se}$ in different cases.
longitudinal resistance of SCCN. The effect of $R_s$ on $I_{rs}$ and $\eta$ are shown in Fig. 13. According to it, with the increase of $R_s$, the change of $I_{rs}$ is weeny, less than 0.50 A in two cases. Whereas, $R_s$ has a great impact on $\eta$. When $R_s$ increases, $\eta$ decreases significantly. When $R_s < 0.05 \, \Omega \cdot \text{km}^{-1}$, $\eta$ can be more than 81.48% under Case1. Reducing the longitudinal resistance of SCCN can effectively improve the efficiency of collecting stray current of SCCN. While the efficiency of Case2 reduces by more than half compared with Case1.

V. SITE TEST

A. Test Description

The test is carried out in Chengdu Metro Line 1 under the situation of SCDD working and non-working. The Chengdu Metro Line 1 is a 41km and DC 1500V line.

At non-operating time, a single train departs from TPS A, passes through TPS B, arrives at station C. After that, the train turns back from C to A in the same track. The train stops at each station for a few seconds. The train runs on the same route twice according to the same driver’s operation. In the first trip, the SCDDs of the TPSs do not work. In the second trip, the SCDD in TPS B works. The TPS A locates at the head of the line. The TPS B is 1.53km away from TPS A.

During the test, only TPS A and TPS B supply power. The synchronous acquisition devices of 16 channels are used to monitor the feeder current, rail potential (the sum of $u_{rs}$ and $u_{se}$), main circuit current of the SCDD and earth branch current of the SCDD, as shown in Fig. 14. And Fig. 15 shows the cabinet of SCDD, including the current limiting resistor and diode. LEMs sensor are clamped on the branch cables to measure the main circuit current of the SCDD and the earth branch current of the SCDD. The monitoring signals at different locations are synchronized by 4G network. The on-board operating recording system is used to measure the current of the train and traction network potential.

B. Test Result and Analysis

When the train runs between B and C, the current of the train on the two trips is shown in Fig. 16 and Fig. 17. Rail potential of A and B are shown in Fig. 18 and Fig. 19. The current of the SCDD in TPS B in the second trip is shown in Fig. 20.

As shown in Fig. 16 and Fig. 17, there are two acceleration processes from B to C and from C to B. The train’s regenerative energy is mainly consumed by on-board braking resistors. In the first trip, the train’s current is 1389A at $t_{11} = 02:58:16$ and 1359A at $t_{12} = 03:01:28$. In the second trip, the train’s current is 1306A at $t_{21} = 03:10:36$ and 1229A at $t_{22} = 03:13:44$. Little difference between the two trips may due to the driver’s operation and the change of circuit structure caused by SCDD.

As shown in Fig. 18 and Fig. 19, when the train stops at station C, the rail potential is almost zero and the fluctuation is very small. When the train runs between B and C, no regenerative energy is recovered.
Fig. 17. Train current in the second trip when the train runs from B to C and returns to B.

Fig. 18. Rail potential of TPS A and B in the first trip when the train runs from B to C and returns to B.

Fig. 19. Rail potential of TPS A and B in the second trip when the train runs from B to C and returns to B.

Fig. 20. Current of SCCN in TPS B in the second trip when the train runs from B to C and returns to B.

Fig. 21. Stray current distribution when the SCDD working.

Energy feedback to overhead catenary during the train’s braking process. The rail potentials at TPS A and TPS B are negative. Moreover, TPS A is further away from the train than TPS B. Part of the train current returns to TPS B, and the remaining of the current returns to TPS A. The superposition of rail potential generated by these two parts of current makes the potential of TPS A more negative.

In Fig. 19, because of the working of SCDD in TPS B, its low resistance path reduces the potential drop between the rail and the earth at TPS B, the rail potential along the rail rises. The amplitude of rail potential in Fig. 19 is significantly smaller than that in Fig. 18. In the first trip, the rail potential of TPS A is -10.207V at $t_{11}$ and -31.471 V at $t_{12}$. The rail potential of TPS B is -6.025 V at $t_{11}$ and -23.507 V at $t_{12}$. In the second trip, the rail potential of TPS A is -8.123 V at $t_{21}$ and -9.312 V at $t_{22}$. The rail potential of TPS B is -2.520 V at $t_{21}$ and -9.312 V at $t_{22}$.

The peaks of the main circuit current and the earth branch current of the SCDD correspond to the valleys of the rail potential of TPS A and B, and correspond to the peaks of the traction current of the train. At $t_{21}$ and $t_{22}$, the proportion of earth branch current of the SCDD to main circuit current is 94.32%, 96.46% respectively.

C. Model Reproducing

The stray current cannot be evaluated directly, which relates to the rail potential closely. The rail reflux system model can be utilized to analyze the test by combined with practical conditions and reflux parameters. The research group measured the $R_{rs}$ of the section, and the result is 4.10 $\Omega$·km. The longitudinal resistances of SCCN (12.5m ballast bed) are spot checked. The longitudinal resistance of single ballast bed is between 1m $\Omega$ and 2.5m $\Omega$, and the connection of cables between ballast beds will also increase $R_s$. So $R_s$ is taken as 0.20 $\Omega$/km. $R_{se}$ is a difficult parameter to be determined, and needs to be solved by fitting $g$.

Fig. 21 shows the path of stray current diffusion in the second trip. The primary and secondary stray currents between 0 km and 1.53km is small enough to be neglected. Then, the main circuit current and the earth branch current of the SCDD can be regarded as $I_{rs}$, $I_{se}$ respectively. $g$ is about 5.68% and 3.54% of test result at $t_{21}$ and $t_{22}$.

Define the $\rho_1$ and $\rho_2$ are the error of efficiency between the test result and calculation results at $t_{21}$ and $t_{22}$. Each of them has a weight value of 0.5. The comprehensive error $\rho$ is defined as (28). Result is shown in Fig. 22. As a summary, $\rho$ changes from positive to negative with the increase of $R_{se}$. When $R_{se} = 0.23\Omega$·km, $\rho$ is the smallest at $t_{21}$ and $t_{22}$, which is 0.03%. At last, in the rail reflux system model, $R_{se}$ is set as 0.23 $\Omega$·km.

$$\rho = 0.5\rho_1 + 0.5\rho_2$$  \hspace{1cm} (28)

The calculation value of rail potential of TPS A and TPS B is compared with the test as shown in Fig. 23 and Fig. 24. The error is defined as absolute value of difference between test value and calculated value.
The maximum error of rail potential between calculation and test is less than 0.1V. The results of model calculation are reliable.

The rail reflux system model is used to calculate the rail potential distribution. The rail potential distribution of the first trip and second trip are shown in Fig. 25 and Fig. 26, and the difference of rail potential distribution ($\Delta U_{re}$) between the second trip and the first trip is shown in Fig. 27.

In the first trip, when the TPS A has the lowest rail potential -31.473V, the train has the highest rail potential 3.483V. Furthermore, the range of anode area is wider than that of cathode area. In the second trip, the lowest rail potential is -22.209V, which is 9.266V lower than the first trip. The highest rail potential is 14.323V, which is 10.840V higher than the first trip. From Fig. 27, $\Delta U_{re}$ is positive generally, which means that the SCCD makes the $U_{re}$ rise most of the time. $\Delta U_{re}$ is negative between 0 to 1.6km at few seconds, which may be caused by the difference between the train current and the traction substation current.

According to the calculation results, the primary stray current and the efficiency of collecting stray current of SCCN are shown in Fig. 28 and Fig. 29.

From Fig. 28, in the C-B process of the first trip, the average primary stray current is 3.14 A. The average primary stray current increases 1.35A compared with B-C process as a result of longer power supply distance in the C-B accelerated process.

As Fig. 29 shows, the average primary stray current is 17.58 A in the C-B process. And it increases 5.60 times compared with the first trip. Corresponding, the average second stray current are 2.68 A and 16.87A in the C-B process of the first trip and second
trip. In other words, the SCDD working significantly leads to the increase of $I_{rs}$ and $I_{sc}$.

As Fig. 28 shows, in the first trip, the $\eta$ depends on the location of the train. The $\eta$ is near 17% when the train is close to TPS B and 13% when the train is far away from B. The average $\eta$ is 12.73% and 10.87% in B-C process and C-B process. The reasons for the low efficiency are as follows: the longitudinal resistance of SCCN of a single track bed block is larger than the design value. Cable bolts with a cross-sectional area of 95mm$^2$ are used to connect the reinforcement cage of the short track bed block, and the connections also increase $R_c$. The insulated through bare conductor and the reinforcement cage of the track bed are welded as the SCCN to enhance the electrical connectivity [8]. Whether this is a significant mean to reduce the longitudinal resistance is worth studying. In addition, it is necessary to strengthen the insulation installation performance of SCCN during the design and construction to enhance the $R_{sc}$.

As shown in Fig. 29, the average $\eta$ is only 4.01% and 3.27% in B-C process and C-B process. In a word, the primary stray current and the secondary stray current increases significantly after the SCDD working.

VI. CONCLUSION

In this paper, a comprehensive system integrated calculation model includes iterative power flow algorithm with simplification of DTS and multi sections rail reflux system model is proposed. The model is applied to analyze the effect of stray current collection system on stray current. The evaluation of the effect of stray current mainly depends on three indexes, the primary stray current ($I_{rs}$), the secondary stray current ($I_{sc}$), and efficiency of collecting stray current of SCCN($\eta$).

The stray current control effect by using the SCDD is analyzed. The working of the SCDD clamps the potential of traction power substation, which greatly increased the rail potential along the line. And the anode area and primary stray current increases. Moreover, the increase of secondary stray current is larger than the primary stray current, and the efficiency of collecting stray current by stray current collection network is reduced. CDEGS simulation and site test verify the accuracy of the proposed model.

The influence of stray current collection system parameters is also discussed by the model. The results show that $I_{rs}$ increases with the decrease of rail to SCCN resistance and the increase of the length of the line. The $\eta$ is reduced with the increase of longitudinal resistance of the SCCN.

REFERENCES


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